

On the charged meson pair theory of nuclear forces

Part II

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With 1 figure in the text

Chapter III

Applications on the nuclear forces

In this chapter¹ we shall discuss whether the known experimental facts of the nuclear forces can be accounted for by a potential deduced only from a charged meson field and particularly how far a potential deduced from a charged meson pair theory alone may be sufficient. Before treating the general problem we shall look in some detail into the problems of the interaction between protons. This is of special interest as, with the exception of the COULOMB force, the forces between two protons must in the first approximation be given entirely by the pair theory.

§ 1. The differential equations

We shall at first write down the differential equations for the two nucleon problem assuming the velocity of the nucleons to be so small that we may neglect terms proportional to it. We shall later discuss whether this is a permissible approximation. This means that we may neglect all interaction terms containing $S_1(\mathbf{r})$, $\mathbf{S}_1(\mathbf{r})$, $S_2(\mathbf{r})$, and $\mathbf{S}_2(\mathbf{r})$ being essentially proportional to the velocity and thus small compared with $S_0(\mathbf{r})$, $\mathbf{S}_0(\mathbf{r})$, $S_3(\mathbf{r})$, and $\mathbf{S}_3(\mathbf{r})$. Further, as in this approximation ϱ_3 may be put $= -1$, we have $S_3(\mathbf{r}) = S_0(\mathbf{r})$ and $\mathbf{S}_3(\mathbf{r}) = \mathbf{S}_0(\mathbf{r})$. The interaction potential in Chapter I then can in the usual way be seen to correspond to an interaction potential W in the ordinary treatment of the two nucleon problem ($S_0(\mathbf{r})$ corresponds to $\sum_{v=1}^2 \delta(\mathbf{r} - \mathbf{r}_v)$ and $\mathbf{S}_0(\mathbf{r})$ to $\sum_{v=1}^2 \boldsymbol{\sigma}_v \delta(\mathbf{r} - \mathbf{r}_v)$, $\mathbf{r}_v$ giving the position of the nucleon v) which can be written

$$W = \frac{32}{\pi} \left(\frac{e^2}{\hbar c} \right)^2 \mu c^2 \{ W_1(2\pi r) + \boldsymbol{\sigma}_1 \boldsymbol{\sigma}_2 W_2(2\pi r) + S_{12} W_3(2\pi r) \} \quad (1)$$

¹ Chapter I and II of this paper are published in Arkiv f. fysik bd 1 nr 4.

where the "tensor force operator"

$$S_{12} = 3 \frac{(\sigma_1 \mathbf{r})(\sigma_2 \mathbf{r})}{r^2} - \sigma_1 \sigma_2$$

and

$$\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1.$$

In the case of scalar mesons¹ we get (Chapter I, 70)

$$W_{1s}(z) = C'_{s1} \left[-\frac{0.5}{z^2} K_1(z) \right] + \quad (S_{1'1'}) \quad (2)$$

$$+ C''_{s1} \left[-\left(\frac{240}{z^6} + \frac{42}{z^4} + \frac{0.5}{z^2} \right) K_1(z) - \left(\frac{120}{z^5} + \frac{6}{z^3} \right) K_0(z) \right] + \quad (S_{1''1''})$$

$$+ C'_{s1} C''_{s1} \left[-\left(\frac{12}{z^4} + \frac{1}{z^2} \right) K_1(z) - \frac{6}{z^3} K_0(z) \right] + \quad (S_{1'1''})$$

$$+ C'^2_{s2} \left[\frac{4}{z^4} K_1(z) + \frac{2}{z^3} K_0(z) \right] \quad (S_{2'2'})$$

$$W_{2s}(z) = \frac{1}{3} C'^2_{s3} \left[\left(\frac{160}{z^6} + \frac{28}{z^4} \right) K_1(z) + \left(\frac{80}{z^5} + \frac{4}{z^3} \right) K_0(z) \right] \quad (S_{3'3'}) \quad (3)$$

$$W_{3s}(z) = \frac{1}{3} C'^2_{s3} \left[-\left(\frac{140}{z^6} + \frac{20}{z^4} \right) K_1(z) - \left(\frac{70}{z^5} + \frac{2}{z^3} \right) K_0(z) \right]. \quad (S_{3'3'}) \quad (4)$$

In the vector case² we get (Chapter I (67)) (5)

$$W_{1v}(z) = \left\{ C'^2_{v1} \left[-\left(\frac{240}{z^6} + \frac{42}{z^4} + \frac{1.5}{z^2} \right) K_1(z) - \left(\frac{120}{z^5} + \frac{6}{z^3} \right) K_0(z) \right] + \quad (V_{1'1'}) \right.$$

$$+ C''_{v1} \left[-\left(\frac{480}{z^6} + \frac{84}{z^4} + \frac{1.5}{z^2} \right) K_1(z) - \left(\frac{240}{z^5} + \frac{12}{z^3} \right) K_0(z) \right] + \quad (V_{1''1''})$$

$$+ C'_{v1} C''_{v1} \left[\left(\frac{36}{z^4} + \frac{3}{z^2} \right) K_1(z) + \frac{18}{z^3} K_0(z) \right] + \quad (V_{1'1''})$$

$$+ C'^2_{v2} \left[\left(\frac{320}{z^6} + \frac{68}{z^4} \right) K_1(z) + \left(\frac{160}{z^5} + \frac{14}{z^3} \right) K_0(z) \right] + \quad (V_{2'2'})$$

$$+ C''_{v2} \left[\left(\frac{320}{z^6} + \frac{56}{z^4} \right) K_1(z) + \left(\frac{160}{z^5} + \frac{8}{z^3} \right) K_0(z) \right] \left. \right\} \quad (V_{2''2''})$$

¹ (S_{1'1'}) has been calculated by WENTZEL [41], a part of (S_{1''1''}) by PAULI and HU [45], (S_{3'3'}) by JAUCH and LOPEZ [44] (cf. a correction in PAULI and HU).

² (V_{3'3'}) is calculated by KLEIN [43] and (V_{2'2'} + V_{3'3'} + V_{3'3''} + V_{3'3''}) with equal C's by BRULIN and HJALMARS [45].

$$\begin{aligned}
 W_{2v}(z) = & \frac{1}{3} \left\{ C'_{v3} \left[\left(\frac{160}{z^6} + \frac{32}{z^4} - \frac{1}{z^2} \right) K_1(z) + \left(\frac{80}{z^5} + \frac{6}{z^3} \right) K_0(z) \right] + \right. & (V_{3'3'}) \\
 & + C'_{v3} \left[\left(\frac{160}{z^6} + \frac{32}{z^4} - \frac{1}{z^2} \right) K_1(z) + \left(\frac{80}{z^5} + \frac{6}{z^3} \right) K_0(z) \right] + & (V_{3''3''}) \\
 & + C'_{v3} C''_{v3} \left[\left(\frac{32}{z^4} + \frac{2}{z^2} \right) K_1(z) + \frac{16}{z^3} K_0(z) \right] + & (V_{3'3''}) \quad (6) \\
 & + C'_{v4} \left[- \left(\frac{640}{z^6} + \frac{56}{z^4} \right) K_1(z) - \left(\frac{320}{z^5} - \frac{12}{z^3} \right) K_0(z) \right] + & (V_{4'4'}) \\
 & + C'_{v4} \left[- \left(\frac{640}{z^6} + \frac{160}{z^4} + \frac{8}{z^2} \right) K_1(z) - \left(\frac{320}{z^5} + \frac{40}{z^3} \right) K_0(z) \right] \Big\} & (V_{4''4''})
 \end{aligned}$$

and

$$\begin{aligned}
 W_{3v}(z) = & \frac{1}{3} \left\{ C'_{v3} \left[- \left(\frac{560}{z^6} + \frac{100}{z^4} + \frac{1}{z^2} \right) K_1(z) - \left(\frac{280}{z^5} + \frac{15}{z^3} \right) K_0(z) \right] + \right. & (V_{3'3'}) \\
 & + C'_{v3} \left[- \left(\frac{140}{z^6} + \frac{40}{z^4} + \frac{1}{z^2} \right) K_1(z) - \left(\frac{70}{z^5} + \frac{9}{z^3} \right) K_0(z) \right] + & (V_{3''3''}) \\
 & + C'_{v3} C''_{v3} \left[\left(\frac{20}{z^4} + \frac{2}{z^2} \right) K_1(z) + \frac{10}{z^3} K_0(z) \right] + & (V_{3'3''}) \quad (7) \\
 & + C'_{v4} \left[\left(\frac{560}{z^6} + \frac{100}{z^4} \right) K_1(z) + \left(\frac{280}{z^5} + \frac{12}{z^3} \right) K_0(z) \right] + & (V_{4'4'}) \\
 & + C'_{v4} \left[\left(\frac{560}{z^6} + \frac{140}{z^4} + \frac{4}{z^2} \right) K_1(z) + \left(\frac{280}{z^5} + \frac{32}{z^3} \right) K_0(z) \right] \Big\}. & (V_{4''4''})
 \end{aligned}$$

The analysis of the two nucleon problem with a potential of the type⁽¹⁾ has been performed by a great many authors (see e. g. KITTEL and BREIT [39], BREIT, KITTEL and THAXTON [40], BETHE [40] cf. ROSENFELD [48], pp. 53 and 333). A state is defined by the total spin (quantum number $S=0$ or 1), the total angular momentum and its component in one direction (quantum numbers j and m respectively) and by the parity of the state. $S=0$ corresponds to a singlet state, $S=1$ to a triplet state. Using the usual spectroscopic notation ${}^aL_j$ we get:

j	Symmetric states ¹		Antisymmetric states ²	
	$S=0$	$S=1$	$S=0$	$S=1$
0			${}^1S_0$	${}^3P_0$
1	${}^1P_1$	${}^3S_1 + {}^3D_1$		${}^3P_1$
2		${}^3D_2$	${}^1D_2$	${}^3P_2 + {}^3F_2$

(8)

¹ States symmetric or antisymmetric in both space and spin coordinates.

² States symmetric in space and antisymmetric in spin coordinates or vice versa.

In the barycentric system the wave function of the nucleons $\psi(\mathbf{r})$ is determined by

$$\left(\frac{\hbar^2}{M} \Delta - W + E_b\right) \psi(\mathbf{r}) = 0 \quad (9)$$

where E_b is the energy in the barycentric system. To the potential W from (2)–(7) another potential may possibly be added. This will then be of the type given in (1) or of the type (1) multiplied by an operator exchanging neutrons and protons. For example, in the case of interaction between protons we shall have to add the COULOMB potential. We put

$$\psi(\mathbf{r}) = \sum_{l'} {}^\sigma Z_j^{(l')m} (\vartheta \varphi)^\sigma R_j^{j-l'}(r) \frac{1}{r} \quad (10)$$

where $Z(\vartheta, \varphi)$ are given in ROSENFELD [48] p. 56.

For a system consisting of two protons we get the following differential equations. If we assume one proton to be at rest and the other to have the kinetic energy E we have

$$E = 2 E_b.$$

We put further

$$z = 2\pi r \text{ as above, } k = \frac{1}{2\pi\hbar} \sqrt{\frac{ME}{2}}, \quad \alpha = \frac{e^2}{\hbar} \sqrt{\frac{M}{2E}}, \quad \beta = \frac{8}{\pi} \left(\frac{e^2}{\hbar c}\right)^2 \frac{M}{\mu}.$$

Then we get in the barycentric system

$$\begin{aligned} S=0 \quad (\sigma=1) \quad l'=j \\ \left[\frac{d^2}{dz^2} - \frac{j(j+1)}{z^2} - \frac{2ka}{z} - \beta(W_1(z) - 3W_2(z)) + k^2 \right] {}^1R_j^0(z) = 0 \end{aligned} \quad (11)$$

$$\begin{aligned} S=1 \quad (\sigma=3) \quad l'=j \\ \left[\frac{d^2}{dz^2} - \frac{j(j+1)}{z^2} - \frac{2ka}{z} - \beta(W_1(z) + W_2(z) + 2W_3(z)) + k^2 \right] {}^3R_j^0(z) = 0 \end{aligned} \quad (12)$$

$$\begin{aligned} S=1 \quad (\sigma=3) \quad l'=j-1 \quad \text{and} \quad l'=j+1 \\ \left[\frac{d^2}{dz^2} - \frac{j(j-1)}{z^2} - \frac{2ka}{z} - \beta \left(W_1(z) + W_2(z) - 2\frac{j-1}{2j+1} W_3(z) \right) + k^2 \right] {}^3R_j^1(z) + \\ + \beta \frac{6Vj(j+1)}{2j+1} W_3(z) {}^3R_j^{-1}(z) = 0 \\ \left[\frac{d^2}{dz^2} - \frac{(j+1)(j+2)}{z^2} - \frac{2ka}{z} - \beta \left(W_1(z) + W_2(z) - 2\frac{j+2}{2j+1} W_3(z) \right) + \right. \\ \left. + k^2 \right] {}^3R_j^{-1}(z) + \beta \frac{6Vj(j+1)}{2j+1} W_3(z) {}^3R_j^1(z) = 0 \end{aligned} \quad (13)$$

If anyone of the particles is a neutron instead of a proton the term $\frac{2ka}{z}$ disappears. It must be observed that in the case of protons alone (or neutrons alone) the symmetric states are missing.

§ 2. Scattering of protons by protons

The most important experimental data known on the interaction between protons are given by the scattering experiments. Owing to the assumptions in the introduction to this chapter it should be possible to account for these data even in detail by a potential deduced from the meson pair theory. It may thus be of interest to calculate the cross sections for protons of different energies scattered by protons, assuming a particular choice of the interaction constants. In this way we get a comparison between the meson pair potential and the experimental results. We shall use the potential $V_{3''3''}$ of (6) and (7). To get convergent results (cf. § 3) the meson pair potential has been cut off and put $= 0$ at some "cut-off distance". To make a comparison with the neutron-proton interaction possible, the COULOMB potential has been retained even inside this distance. As two parameters, the interaction constant C^1 and the cut-off distance z_0 , may be chosen arbitrarily, we must use the values given experimentally for the scattering cross sections for two energies, in order to determine the potential.

In a paper by HJALMARS and myself² a calculation analogous to this was performed. The accuracy of the calculations there was such that an error of up to 5 % in the determination of the cut-off distance for a given value of the interaction constant was possible. This error was too large to permit a determination of the two parameters as accurately as was desirable for comparison with the experiments. Thus in the present paper corresponding calculations have been performed using another method of integration and with greater accuracy. Except in one case (an energy of 10 MeV), the energy of the protons is so small that all states higher than the S state may be neglected. In this particular case the cross sections of the P states are calculated too. As small P phase shifts are roughly proportional to $E^{3/2}$, it follows immediately from the values of these phase shifts that it is permissible to neglect as mentioned above. The differential equation of the ^1S state is given by (11) when $j=0$, and the attractive potential $-3\beta W_2$ is given by (6) $V_{3''3''}$ (cf. B. H. equ. 5). The P state — being a triplet state — is given by three differential equations. The $^3P_2$ state is coupled with a $^3F_2$ state but the coupling is rather weak and may be neglected. $^3P_0$ is given by the second equation of (13) when $j=0$ and the repulsive potential $\beta(W_2 - 4W_3)$, $^3P_1$ is given by (12) when $j=1$ and the attractive potential $\beta(W_2 + 2W_3)$ and $^3P_2$ is given by the first equation of (13) when $j=2$, neglecting the term of coupling, and the repulsive potential $\beta(W_2 - \frac{2}{5}W_3)$. Using the notation of B. H. we have $W_2 = \frac{1}{3}C^2(3A - B)$ and $W_3 = -\frac{1}{3}C^2B$ where A and B are given in B. H. eqs. (2) and (3). For the integration in the ^1S state we have used the

¹ In this section we shall drop the indices of C''_{v3} .

² BRULIN and HJALMARS [45]. Here quoted as B. H.

asymptotic solutions (12) in B.H.¹ and in the P state the corresponding asymptotic solutions (cf. Chapter II § 1 A; the first term of the expansion is given in (5 A) with $l = 1$). In the numerical integrations we have used a method, developed by HARTREE [33]. Two fundamental solutions were calculated, one with the phase

$$\eta = \begin{cases} \eta_0 = \arg \Gamma(1 + i\alpha) \text{ in the } {}^1S \text{ state} \\ \eta_1 = \arg \Gamma(2 + i\alpha) \text{ in the } {}^3P \text{ state respectively} \end{cases} \quad (\text{solution A}) \quad (14)$$

(cf. Chapter II § 1 A where the expansion of $\arg \Gamma$ is given) and the other with the phase

$$\eta = \begin{cases} \eta_0 + \frac{\pi}{2} \\ \eta_1 + \frac{\pi}{2} \text{ respectively} \end{cases} \quad (\text{solution B}) \quad (15)$$

(A) is calculated down to zero with

$$W_1 = W_2 = W_3 = 0 \quad (Ae).$$

In the calculations the pair potential could be neglected down to $z \approx 7$. At $z \approx 4$, where the pair potential begins to be predominant, the differential equations were transformed using B.H. equ. (7) when $n = 1$ i. e. we put $y = \frac{1}{z}$ and $F(y) = R(z) \cdot z^{-1}$, thus getting a less singular potential. Then we continued the integration down to the cut-off distance. As is well known the wave function must be zero at the origin and, thus, (Ae) is the correct solution inside the cut-off distance. To get a smooth connection between the solutions the ratios between their derivatives and themselves must be equal. If the wave function outside the cut-off distance is $R(z)$ ($R_A(z)$ in case (A), $R_B(z)$ in (B), $R_{Ae}(z)$ in (Ae)) and the phase shift K we obtain

$$R(z) = R_A(z) \cos K + R_B(z) \sin K. \quad (16)$$

Putting

$$\frac{R'(z)}{R(z)} = w(z) \quad (17)$$

¹ If some errors — too small to be of any significance in B.H. — in the highest terms of (12) are corrected and another term is calculated we get the highest terms of the factor of the sine term.

$$- \frac{12\alpha - 51\alpha^3 + 9\alpha^5}{48k^3z^3} - \frac{564\alpha^2 - 769\alpha^4 + 106\alpha^6 - \alpha^8}{384k^4z^4}$$

and of the cosine term

$$- \frac{40\alpha^2 - 31\alpha^4 + \alpha^6}{48k^3z^3} + \frac{144\alpha - 904\alpha^3 + 376\alpha^5 - 16\alpha^7}{384k^4z^4}$$

and

$$\frac{R'_{Ae}(z)}{R_{Ae}(z)} = w_e(z) \quad (18)$$

we shall have

$$w(z_0) = w_e(z_0). \quad (19)$$

From (16) and (19) we get

$$K = -\arctg \frac{R'_A(z_0) - w_e(z_0) R_A(z_0)}{R'_B(z_0) - w_e(z_0) R_B(z_0)}. \quad (20)$$

We put $K = K_0$ in the ^1S state and $= \delta_0, \delta_1$, and δ_2 in the $^3P_0, ^3P_1, ^3P_2$ states respectively. For two different values of the energy, K_0 was obtained from experimental data. At a value of z where the meson potential is negligible, $R(z)$ was determined from K_0 and (16) and the equation was integrated for some values of C . We plotted $w(z)$ and $w_e(z)$ against z in a diagram for these values of C and determined at which values of z the curves cut each others, thus getting C as a function of z_0 . As the interaction constant of the potential C and the cut-off distance z_0 must be constant independent of the energy of the protons and thus the same in both these cases, we can easily determine them by equating these two functions.

The scattering cross section per unit angle¹ in the barycentric system is given by

$$\sigma = \sigma_M + \sigma_0 + \sigma_1 \quad (21)$$

where the COULOMB scattering cross section (MOTT's formula) is

$$\sigma_M = \frac{1}{16\pi^2} \frac{\alpha^2}{k^2} \left\{ \frac{1}{s^4} + \frac{1}{c^4} - \frac{1}{s^2 c^2} \cos(\alpha \log s^2 c^{-2}) \right\} = \frac{1}{16\pi^2} \frac{\alpha^2}{k^2} M \quad (22)$$

the S scattering cross section is

$$\begin{aligned} \sigma_0 = \frac{1}{16\pi^2} \frac{\alpha^2}{k^2} & \left\{ -\frac{2}{\alpha} \left(\frac{\cos \alpha_0}{s^2} + \frac{\cos \beta_0}{c^2} \right) \sin K_0 \cos K_0 + \right. \\ & \left. + \left[\frac{4}{\alpha^2} + \frac{2}{\alpha} \left(\frac{\sin \alpha_0}{s^2} + \frac{\sin \beta_0}{c^2} \right) \right] \sin^2 K_0 \right\} \end{aligned} \quad (23)$$

and the P scattering cross section is

$$\begin{aligned} \sigma_1 = \frac{1}{16\pi^2} \frac{\alpha^2}{k^2} & \left\{ \sum g_i \left[-\frac{18}{\alpha} \cos \theta \left(\frac{\cos \alpha_1}{s^2} - \frac{\cos \beta_1}{c^2} \right) \sin \delta_i \cos \delta_i + \right. \right. \\ & \left. + \left(\frac{108}{\alpha^2} \cos^2 \theta + \frac{18}{\alpha} \cos \theta \left(\frac{\sin \alpha_1}{s^2} - \frac{\sin \beta_1}{c^2} \right) \right) \sin^2 \delta_i \right] - \\ & - \frac{1}{\alpha^2} (3 \cos^2 \theta - 1) \left[9 (\sin \delta_1 - \sin \delta_2)^2 + 4 (\sin \delta_0 - \sin \delta_2)^2 + \right. \\ & \left. + 36 \sin \delta_1 \sin \delta_2 \sin^2 \frac{\delta_1 - \delta_2}{2} + 16 \sin \delta_0 \sin \delta_2 \sin^2 \frac{\delta_0 - \delta_2}{2} \right] \Big\} \end{aligned} \quad (24)$$

¹ cf. BREIT, CONDON and PRESENT [36] and BREIT, KITTEL and THAXTON [40].

where θ is the angle between the incident and the scattered protons in the barycentric system; the weights $g_0 = \frac{1}{9}$, $g_1 = \frac{3}{9}$ and $g_2 = \frac{5}{9}$; $s = \sin \frac{\theta}{2}$ and $c = \cos \frac{\theta}{2}$; $\alpha_0 = \alpha \log s^2$, $\beta_0 = \alpha \log c^2$, $\alpha_1 = \alpha_0 + 2(\eta_1 - \eta_0)$, $\beta_1 = \beta_0 + 2(\eta_1 - \eta_0)$.

For different values of θ and different energies, $\mathfrak{M}$ is given in BREIT, THAXTON and EISENBUD¹ [39] table V. The coefficients of the different trigonometrical functions divided by $\mathfrak{M}$ up to an energy of 10 MeV are given in BREIT [36], B. T. E. and THAXTON and HOISINGTON [39]. For the energy, which is of most interest to us (10 MeV), the coefficients of $-\sin K_0 \cos K_0$ is given in table II, of $\sin^2 K_0$ in table III, of $-\sin \delta_i \cos \delta_i$ in table IV and of $\sin^2 \delta_i$ in table V in THAXTON and HOISINGTON [39].

A great many experiments on the scattering of protons by protons have been carried out, and in the energy region from 0.2 to 4 MeV particularly, they are rather accurate and permit definite conclusions. The analysis of these data shows that the potential is attractive and has a range of about $1-3 \cdot 10^{-13}$ cm, and that they can be accounted for by a square well potential. These data have been carefully analysed, the ^1S phase shifts have been calculated and the P phases have been shown to be negligibly small. The estimated error in the phases is 0.4–1 degree, and thus an accuracy in our numerical calculations giving the phase with an error smaller than 0.4 degrees is satisfactory. For a discussion of the experiments and a table of the phase shifts we would refer the reader to ROSENFELD [48] Chapter VII. The analysis of the different experiments shows that they are consistent within the frame of the estimated errors. The experiments of HERB and his collaborators [39] in the region from 0.86–2.4 MeV seem to be particularly reliable, and we have therefore determined the constants of our potential using two phase shifts obtained from their experiments. Numerical calculations have then been performed using these constants, giving as results, in one case, a phase shift to be compared with a third of their values, and in another, with a much lower energy, a phase shift to be compared with a phase shift determined by RAGAN et al. [41] and also consistent with a determination by TUVE et al. [38].

Besides these experiments, a group of experiments in the energy region 7–15 MeV have been carried out by WILSON and collaborators particularly (8 MeV [47 A], 10 MeV [47 B] and 14.5 MeV [47 C and D]). These experiments are not as accurate as those mentioned above and a careful analysis of the data, corresponding to that of the lower energies, cannot be performed. The results indicate, however, that at 10 MeV a small and at 14.5 MeV a somewhat stronger P phase shift must exist in addition to the S phase shifts, and that the P potential must be mainly repulsive. We have calculated the phases at the energy 10 MeV. The experimental errors at 14.5 MeV are so large that a calculation at this energy would not have given a better possibility of comparison with experiments in spite of the larger P scattering. We have thus determined the phase shifts all over the energy region and got good possibilities for comparison with experiments. In the following we shall only cite the experimental values which can directly be compared with our calculated values.

¹ quoted as B. T. E.

For the determination of the constants C and z_0 we chose the phase shifts obtained from the energies 2.392 MeV and 0.860 MeV. These phase shifts were determined by B. T. E. with an error of $0^\circ.4$.

In the calculations we used the same value of the meson mass as in B. H. $\mu = 177 m_e$ (m_e electron mass). We then get

$$\frac{1}{\kappa} = 2.1819 \cdot 10^{-13} \text{ cm}, \quad k = 0.11983 \sqrt{E}, \quad a = \frac{0.15818}{\sqrt{E}},$$

E measured in MeV.

In table I the cut-off distance z_0 is given as a function of C .

Table I.

z_0 is given in units of $\frac{1}{2\kappa}$.

E MeV	K_0	$C =$	5	10	15	20
2.392	$48^\circ.08$	$z_0 =$	1.003	1.311	1.526	1.694
0.860	$29^\circ.28$	$z_0 =$	1.001	1.310	1.528	1.698

Although the differences between the values of z_0 for the same C are rather small the value $C = 5$ is probably too small and $C = 20$ evidently too large. The real value might be somewhere between 10 and 15. The experiments are not sufficiently accurate to make a closer determination possible. We have assumed $C = 10$ to be the best value but have in most cases made calculations for $C = 15$ too. In Table II we shall give the results for different energies.

Table II.

For $C = 10$ we have put $z_0 = 1.311$, $C = 15$ $z_0 = 1.527$.

E MeV	Experimental K_0	$C = 10$	Calculated K_0	$C = 15$
0.300	$10^\circ.7 \pm 0^\circ.5$ ¹	$11^\circ.2$		$11^\circ.6$
0.860	$29^\circ.28 \pm 0^\circ.4$ ²		table I	
1.836	$44^\circ.02 \pm 0^\circ.4$ ²	$43^\circ.9$		$44^\circ.1$
2.392	$48^\circ.08 \pm 0^\circ.4$ ²	$47^\circ.8$ ⁴		table I
10	$52^\circ.5$ ³	$55^\circ.0$		

¹ This value is obtained by interpolation from the experiments of RAGAN et al. [41]. The analysis of the experiments gives for $E = 298.3$ $K_0 = 10^\circ.5$ and $E = 321.4$ $K_0 = 13^\circ.4$. It can be compared with TUVE et al. [38] where $E = 220$ gives $K_0 = 3^\circ.5$ and $E = 325$ gives $K_0 = 14^\circ.1$.

² From HERB et al. [39] analyzed in B. T. E.

³ From WILSON [47 B], the value $52^\circ.5$ is, however, mainly theoretical, being computed on the basis of a square well of range $2.8 \cdot 10^{-13}$ cm by PEIERLS and PRESTON [47] (cf. discussion below).

⁴ This value was computed in the same way as the other to get a check on the accuracy of the calculations. We get an error of $0^\circ.3$, which is well within the uncertainty of the experiments.

The value 0.300 MeV may indicate that C should be chosen somewhat smaller than 10.

For $E = 10$ MeV the P phase shifts were calculated too. We obtained $\delta_0 = -1^\circ.6$, $\delta_1 = 0^\circ.6$ and $\delta_2 = -0^\circ.4$. The scattering cross sections per unit angle (cf. (21), (22) and (23)) are given in Fig. 1, where the experimentally determined cross sections are given too. The mean square errors are indicated. As these values are only relative, we have adjusted them so that the experimental and theoretical cross sections are equal at $\theta = 90^\circ$ ($5.18 \cdot 10^{-26}$ cm²). (Cf. a discussion of these values by PEIERLS and PRESTON [47], FOLDY [47] and RAMSEY [48]).

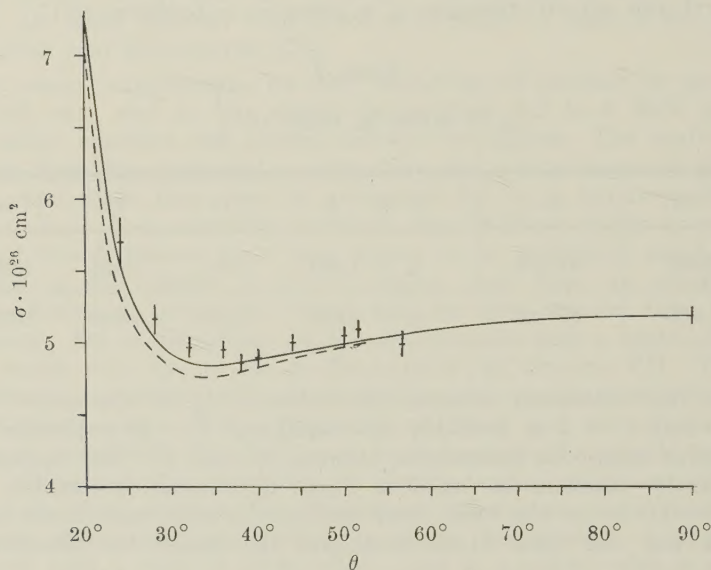


Fig. 1. The cross section per unit solid angle in the barycentric system as a function of the scattering angle in the barycentric system. The crosses give the experimental values. The dotted curve gives the theoretically determined scattering on the basis of COULOMB and S scattering, the solid curve gives the theoretically determined total scattering.

We see from Fig. 1 that we have got the right sign of the P phase shifts and also agreement within the rather large limits of error. Values of the P phase shifts somewhat larger than those obtained by this potential can of course not be excluded by the experiments.

Thus we see that a meson pair potential can, within the actual accuracy of experiments, account for the scattering of protons by protons.

We must, however, investigate, whether the values of C and z_0 are reconcilable with the approximations used in Chapter I. Assuming the "radius" of the nucleon a defined in Chapter II § 2 to be of the same order of magnitude as the cut-off distance and putting $C = 10$ and $\kappa a = \frac{2}{3}$ (Table I) we get from Chapter II (87)

$$2 \frac{C}{(\kappa a)^3} \frac{e^2}{\hbar c} = \frac{1}{2}.$$

This value is to be compared with 1.

We might further (cf. MØLLER and ROSENFELD [40]), estimate whether it is permitted to neglect the gyration of the spin of the nucleons. We have in the first approximation

$$\frac{d\sigma}{dt} = -\frac{i}{\hbar}[\sigma, W]$$

where W is given in (1) and thus

$$\frac{d\sigma_1}{dt} = -\frac{2}{\hbar}\sigma_1 \times \left(\sigma_2(W_2 - W_3) + \frac{\mathbf{r}(\sigma_2 \cdot \mathbf{r})}{r^2} 3W_3 \right) \frac{32}{\pi} \left(\frac{e^2}{\hbar c} \right)^2 \mu c^2. \quad (25)$$

The time of the gyration of the spin must be large compared with the time of the transmission of the field which is at least $= \frac{1}{2\kappa c}$. We may thus define a critical distance z_c such that effects from the gyration of the spins will be important inside it. This distance is given if we put the modulus of the factor of σ equal to $2\kappa c$.

As we are only interested in the order of magnitude of the potential we may put $V_{3''3''} \approx C^2 \frac{200}{z_c^6} K_1(z)$ and thus we get

$$\frac{1}{\hbar} \frac{32}{\pi} \left(\frac{e^2}{\hbar c} \right)^2 \mu c^2 C^2 \frac{200 K_1(z_c)}{z_c^6} \approx 2\kappa c$$

which gives us $z_c \approx 1.2$. z_c is to be compared with the cut-off distance.

As the present theory is founded on simultaneous *virtual* transitions of pairs of mesons, it follows from HEISENBERG's uncertainty principle and from the energy principle that the cut-off distance must not be essentially larger than $\frac{1}{2\kappa}$ (cf. WICK [38]).

We thus see that the conditions for the validity of the calculations in Chapter I are fulfilled but only rather badly and that the potential can only be regarded as a rather rough approximation. The weak coupling approximation seems, however, to be better than the strong coupling one. On the other hand the potential in the strong coupling case for the ^1S state is rather similar to that used above, and thus the results of this section can be regarded as rather well established. From the relatively large value of z_c and from the fact that the velocity dependent terms of $V_{3''3''}$ in Chapter I (67) is only of the second order in the velocity and the coefficients of the same order of magnitude as the velocity independent terms, it follows that these forces must be neglected as far as other terms connected with the motion of the nucleons are neglected. The value of the meson mass used here ($177 m_e$) is probably too low, the masses of the different kinds of mesons produced artificially in the BERKELEY cyclotron being about $210 m_e$ and $285 m_e$ respectively. It is, however, evident that a change in the mass would give mainly the same interaction only making

the potential somewhat steeper in the case of increasing mass. The cut-off distance measured in units of $\frac{1}{2\kappa}$ would, however, be larger for larger mass which is unsatisfactory according to our remarks above.

§ 3. General discussion on the interaction between the nucleons

We shall now discuss the general problem of how far the nuclear interaction can be accounted for by a potential deduced from the meson pair theory. The high singularities of the potentials (in most cases of the order of r^{-7}) and the neglected contact interactions are obviously not reconcilable with any stationary state of a system of nucleons. This means that for small distances the interaction between nucleons cannot be given by a pair theory. As is well known, the same problem occurs at least to some extent even in the ordinary meson theories. The theory has thus to be modified in some way in that region. For the present, however, we shall assume that the potential function is only valid when the distance between the nucleons is larger than some given distance. At this distance the potential will be cut-off and be equated to some other function. This cutting-off procedure is generally performed so that the potential is kept constant inside this distance and equated either to zero ("zero cut-off" used in § 2) or to the value it had at the cut-off distance ("straight cut-off" cf. BETHE [40]). While the value of this distance is crucial for the interaction, the form of the potential function inside it has turned out to be of less importance (BETHE [40], FERRETTI [41]). The magnitude of the cut-off distance depends, of course, on this function. In Chapter II we have discussed the potential assuming a nucleon of finite extension and the radius a defined by (50) of that chapter can as we assumed in § 2 be considered to correspond to the cut-off distance defined above.¹ The high singularities of the potentials deduced from the pair theory together with the fact that the HANKEL functions — being essentially exponential functions — are functions of $2\kappa r$ instead of κr as in the linear theories, imply that the range of the nuclear forces is not very much larger than the cut-off distance. This is an essential difference to the ordinary cut-off theories, implying that the cut-off distance must be chosen larger than in the analogous theories. We have already seen this in § 2. Thus the potential will be rather steep and — particularly if we choose straight cut-off — be more similar to a potential well than to the meson potential.² As it is quite independent of the charge it will have the form of a neutral potential. This means that in the states which are permitted the interaction between two nucleons is the same, independent of the charge of the nucleon. The COULOMB interaction of the protons must of course be added. This is in rather good agreement with experiments. The similarity

¹ If we assume a nucleus to be built up by (A) nucleons of this kind (radius $\approx 1.5 \cdot 10^{-13}$ A¹ cm) we get a value of the radius of the nucleon of the same order of magnitude as the zero cut-off distance ($\approx 1.4 \cdot 10^{-13}$ cm).

² Owing to B. T. E. and THAXTON [42] the square well potential accounts very well for the proton-proton interaction. A potential with "straight cut-off", which resembles both the well and the "zero cut-off" potential, might then account for the proton-proton interaction with a cut-off distance larger than the zero cut-off distance and smaller than the range of the well ($2.8 \cdot 10^{-13}$ cm).

between a charged meson pair potential and a neutral potential implies that the difficulties of a neutral theory will at least to some extent occur even in the pair theory. The lack of forces dependent upon exchange of charge between neutrons and protons makes it doubtful whether it is possible to get saturation in the case of heavier nuclei. As is well known, the current neutral theories do not show saturation. Owing to the modifications of the potential curve necessary inside the rather large cut-off distance, it is, however, not certain that the problem of saturation is quite similar in the pair theory. In fact WENTZEL [45] has shown that it is possible to obtain saturation with an extended source model and an interaction term as those on note 1 p. 147 in Chapter II (cf. HOURIET [43]). Further it may possibly be difficult to account for the scattering of fast neutrons by protons and by deuterons. The experiments on the scattering of neutrons of an energy of 10–15 MeV by protons performed by POWELL and collaborators [44, 46] and by LAUGHLIN and KRUGER [47] give an angular distribution which is almost isotropic but may have a small excess of scattered neutrons in the backward direction in contrast to older experiments by AMALDI et al. [42 A, B] which gave a marked forward scattering. POWELL's and LAUGHLIN's results are at least not consistent with most neutral theories as they have a large P scattering in the forward direction. Experiments on very swift neutrons are rather difficult to interpret in the pair theory because of the effects of the high velocities. From the scattering experiments performed in BERKELEY upon neutrons with energy 90 MeV it does not yet, anyhow, seem possible to draw any definite conclusions as regards these problems. They indicate, however, that at least to some extent the nuclear forces are of exchange type. The interpretation of the scattering of neutrons by deuterons, which however is not unambiguously given by experiments, seem to be difficult to reconcile with a neutral theory (cf. MASSEY and BUCKINGHAM [47]). It seems difficult to account for the many problems of interactions of mesons with nucleons, for the magnetic moments of the nuclei and for the β -radioactivity etc. by a theory only working with the fields of neutral pairs of charged mesons. At the end of this section we shall discuss the possible existence of supplementary forces of other kind deduced from the ordinary charged meson theory (cf. KLEIN [43]). As we said above, the interaction between protons alone or neutrons alone must, however, in the first approximation be accounted for by the pair theory. In the foregoing section we have investigated in some detail the scattering of protons by protons, assuming a particular pair potential. If we choose the parameters in the same way as there, it is, however, evident that — in the ^1S state — any attractive potential or combination of potentials derived from the pair theory will give about the same results, the main features of all these potentials being the same. In the higher states we might get quite different results than in § 2 owing to the fact that a different dependence on the spin can imply other signs and other magnitude of the potentials. It should be kept in mind that the criterion for the validity of the perturbation calculation and the particular assumptions about the nucleons in Chapter I limit the possible choice of the values of the interaction constants when the cut-off distance is fixed. If we further combine the interaction terms so that in one particular state the potential is relatively very small, the velocity dependent terms may be predominant in that state (cf. the Appendix).

We shall now discuss a little more closely the different potential functions which we can obtain from the charged meson pair theory given in (2)–(7) by choosing the interaction constants in different ways. We shall assume the sign and magnitude of the ${}^1S$ potential and the cut-off distance given from the scattering experiments investigated in the foregoing section. This accounts roughly for the scattering of not too fast neutrons by protons too. Besides the problems mentioned above, we shall then have chiefly to consider the ground state and the electric quadrupolemoment of the deuteron.

Let us first regard the possible mainly central potentials. Putting all the interaction constants of § 1 W_2 and $W_3 = 0$ we obtain an entirely spin-independent potential, which we can easily choose so that it is attractive in the ${}^1S$ state, as we have required above. In this case the pair theory does not give any difference between the singlet and triplet states, which difference accordingly has to be totally accounted for by the possible supplementary forces mentioned above. Such a potential will further be attractive in all states of a system of two protons. This is possibly in contrast to the experiments on the scattering of fast protons by protons. As we saw above these experiments indicate that the potential of the P state is weakly repulsive rather than attractive. The potential $S_{11'}$, which is pleasant as it does not have any worse singularities than those occurring in the ordinary cut-off theories, is of this kind. The interaction constant required in this case will, however, be too large to permit the perturbation calculation to be used (cf. WENTZEL [41, 42], JAUCH and LOPES [44]).

If we choose the following relations between the constants of the spin-dependent interaction terms of the vector mesons:

$$C'_{v3} : C''_{v3} : C'_{v4} : C''_{v4} = 2 : 2\sqrt{2} : (\sqrt{2} + 1) : (\sqrt{2} - 1) \quad (26)$$

the tensor force terms cancel each other altogether and we obtain a spin-dependent central force. The $\sigma_1 \sigma_2$ term is then negative. Choosing the constants of the spin-independent terms suitably we can in this way obtain a potential similar to the ordinary neutral central potential, which can account for the ${}^1S$ and ${}^3S$ states. The weak mixture of the ${}^3S$ and ${}^3D$ states necessary to account for the quadrupolemoment of the deuteron must then be obtained from an additional non-central force, which may arise from the velocity-dependent part of the potential or from the supplementary potential mentioned above. The forces in the higher states of the two-proton system will, however, be attractive in this case too, which is, as we saw above, probably not permitted. They will moreover be relatively stronger than in the foregoing case.

We shall now discuss the pair potential where we have retained the tensor force term. In analogy to BETHE's neutral theory [40], KLEIN [43] proposed $V_{3:3''}$ as the potential (cf. also MARSHAK [40]). This potential is mainly attractive in the ${}^1S$ state. In the ground state of the deuteron the sign of the tensor force corresponds to a spatial density distribution which is prolate in respect to the direction of the spins and thus gives the right sign for the quadrupolemoment. A closer investigation shows, however, (B. H.), that the singlet potential is too strong compared with the triplet one and it is therefore impossible to account for both the ${}^1S$ and ${}^3S$ states with the same interaction constant and cut-off distance, as we have tacitly assumed. Thus we

have to combine it with another potential. Without altering the essential features of the potential we succeeded in getting an ^1S potential which is much weaker relatively to the ^3S potential by using the combination $V_{2'2'} + V_{3'3'} + V_{3''3''} + V_{3'3''}$. Assuming that all interaction constants were equal, a numerical investigation (BRULIN and HJALMARS [46]) showed that it was possible to account for the rough features of the scattering experiments and of the ground state simultaneously. We obtained of course the right sign of the quadrupole moment but its value was only about half the experimental one. A more accurate investigation of the scattering of protons by protons showed, however, that the cut-off distance obtained with that potential is too small being about $\frac{11}{4}\pi$ while the calculation in § 2 of this chapter showed that the cut-off distance

in the pair theory must be of the order of magnitude of $\frac{21}{3\pi}$ to account for the details of the proton-proton scattering experiments. But by a simple change of the proportions between the interaction constants we can get a larger value of the cut-off distance. Owing to the similarity with the potential well it is, however, probable that a potential corresponding to the correct value of the cut-off distance will give a quadrupolemoment, which is too large (RARITA-SCHWINGER [41]). It is interesting that in these cases the forces in the P state of two protons are mainly repulsive which is in better agreement with the experiments on fast protons. As we have pointed out before the values of the interaction constants must not be too large if we are to be allowed to use the weak coupling approximation. On the other hand, as has been pointed out by PAULI and HU [45], the forces in the extreme strong-coupling case are spin-independent and thus it is impossible to account for the spin-dependence of the nuclear interaction if the interaction constants are too large.

We have at last the possibility of choosing the constants so that the $\sigma_1 \sigma_2$ term is rather small. If we put $C'_{v3} + C'_{v3} = 4(C'_{v4} + C'_{v4})$ the highest terms of W_{2v} cancel each other. We then get W_{3v} strongly negative. Letting the attractive ^1S potential be given for example by $V_{1'1'}$ we can determine C'_{v1} according to § 2. If we choose the other constants suitably we obtain a potential that is rather similar to the neutral potential used by RARITA-SCHWINGER [41]. (The constants will be ≈ 10). This gives the ground state of the deuteron, but it is probable that, owing to the smaller range, the quadrupolemoment will be too small in this case. The ^3P state will by a choice of constants as that proposed here be mainly attractive and considerably stronger ($\approx$ a factor 10) than in § 2, probably in contrast to experimental results. If however we have to choose the potential more like BETHE's (cf. above) to get a sufficiently large quadrupolemoment, the P potentials will account for the experiments better.

Although perhaps it may be outside the scope of this work we shall finally discuss the possibility of other than pair forces. As was pointed out in the introduction, we shall assume that a possible neutral meson does not play any rôle in the nuclear forces. But other forces might of course exist, as we have pointed out already. It is of course possible that virtual transitions of more than one pair of charged mesons occur simultaneously. Such forces would, of course, be charge-independent too and we may disregard them in the first

approximation. But forces corresponding to transitions of single, charged mesons may exist too. At first the scattering experiments indicate that the ^1S potential is not quite independent of charge but that the neutron-proton potential is some per cent larger than the proton-proton potential. This difference makes it permissible to introduce a weak supplementary attractive force in the ^1S state of the neutron-proton system. These forces might then arise from the ordinary charged meson theory. But we have further the possibility of adding to the former forces charge-dependent forces which without being small are zero in the ^1S state. Such forces might then be either non-central — the tensor force operator is $=0$ in singlet states — or involve the factor $3 + \sigma_1 \sigma_2$.

We can for example in the case of vector mesons obtain a force of that kind by putting the vector interaction constant $g_1 = \sqrt{2}$ times the tensor interaction constant g_2 which gives us a supplementary potential (FRÖHLICH, HEITLER and KEMMER [38]) proportional to

$$-P \left[\left(2 + \frac{2}{3} \sigma_1 \sigma_2 - S_{12} \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{(\kappa r)^2} \right) \right) \right] \frac{e^{-\kappa r}}{r} \quad (27)$$

P is an exchange operator having the value $+1$ in symmetric states, -1 in antisymmetric states (see (8)). In the case of pseudo-vector mesons we can put the pseudo-vector interaction constant $g'_1 = \frac{1}{2}$ times the pseudo-tensor interaction constant g_2 , thus obtaining a supplementary potential (KEMMER [38]) proportional to

$$-P S_{12} \left(\frac{1}{3} + \frac{1}{\kappa r} + \frac{1}{(\kappa r)^2} \right) \frac{e^{-\kappa r}}{r}. \quad (28)$$

As in a pair theory there is no difference between the nuclear forces corresponding to vector or to pseudo-vector mesons (cf. HJALMARS [49]) the potential (28) can just as well be combined with V as the potential (27). It is to be observed that the meson field function U_v in Chapter I is in that case transformed as a pseudo-vector instead of as a vector. Correspondingly, U_s may be obtained from a pseudo-scalar field instead of a scalar field. A mixture of fields of different kinds of mesons may of course give other possibilities. If, for example, we add (27) and (28) we can eliminate the tensor force completely and obtain a supplementary central force attractive in the symmetric triplet states.

It is obviously possible to strengthen the ^3S potential in this way, which is an alternative possibility of getting a sufficiently deep potential in the ground state. From (28), for example, we may obtain a value of the quadrupolemoment of the deuteron which has the correct sign.

These forces will be of the exchange type, which as we saw above is possibly required by the scattering experiments and by the necessity of getting saturation. It is, however, uncertain whether a tensor force as (28) really gives saturation (VOLKOFF [42]). The ordinary interaction terms will, of course, play an important rôle for the theory of the interaction between mesons and matter, of the β -disintegration of the neutrons, of the magnetic moments, etc.

Summing up, we can see from these examples that there exist several possibilities for mixtures between meson pair forces and other forces which might

account for the interaction between neutrons and protons, and thus it is not a crucial point for the pair theory, if the details of a particular experiment concerning neutrons and protons can be given by a particular pair theory. On the other hand, if the pair theory shall be a real alternative to the current meson theories of the nuclear forces, it must be able to account for the interaction between protons alone (the electric forces disregarded) and neutrons alone. This follows from the disappearance of this interaction in the first approximation of a charged meson theory. And as we have seen by an investigation of one particular potential, the main features of the ^1S state of a system of two protons can be accounted for by a potential deduced from a charged meson pair theory. Such a potential can further give the correct sign and order of magnitude of the P phase shifts.

Appendix

We shall show here that with particular mixtures of the pair potentials the velocity dependent terms may be predominant. We shall as an example discuss the potential $V_{2'2'} + V_{3'3'} + V_{3''3''} + V_{3'3''}$ where we have included the terms of the first order in the velocity of the nucleons $V_{2'3'} + V_{2'3''}$. As we saw above, the velocity independent terms of this potential are particularly small in the ^1S state.

We have (cf. ROSENFELD [45], [48] § 15.34) from § 1 and Chapter I (3), (5) and from the equation

$$\begin{aligned} \frac{dF}{dt} &= \frac{1}{i\hbar} [F, H] \\ S_1 &= \sum_{v=1}^2 \frac{1}{2Mc} (\mathbf{p}_v \delta(\mathbf{r} - \mathbf{r}_v) + \delta(\mathbf{r} - \mathbf{r}_v) \mathbf{p}_v) + \frac{\hbar}{2Mc} \text{rot } \mathbf{S}_0 - \frac{\hbar}{2Mc^2} \frac{d\mathbf{S}_2}{dt} \\ S_2 &= \sum_{v=1}^2 \frac{1}{2Mc} \sigma_v \times (\mathbf{p}_v \delta(\mathbf{r} - \mathbf{r}_v) + \delta(\mathbf{r} - \mathbf{r}_v) \mathbf{p}_v) + \frac{\hbar}{2Mc} \text{grad } S_0 + \frac{\hbar}{2Mc^2} \frac{d\mathbf{S}_1}{dt}. \end{aligned} \quad (29)$$

Neglecting the time derivatives, which are of higher order in the velocity, we introduce (29) in Chapter I (67) $V_{2'3'} + V_{2'3''}$ and integrate as in § 1. We get the part of the potential which is of the first order in the velocity

$$\begin{aligned} &= \frac{32}{\pi} \left(\frac{e^2}{\hbar c} \right)^2 \mu c^2 \left\{ \frac{\mu}{M} \left[(-1 - \frac{2}{3} \sigma_1 \sigma_2) \left(f'(2\kappa r) + 2 \frac{f(2\kappa r)}{2\kappa r} \right) + \right. \right. \\ &\quad \left. \left. + \frac{1}{3} S_{12} \left(f'(2\kappa r) - \frac{f(2\kappa r)}{2\kappa r} \right) \right] - \frac{1}{Mc} (\sigma_1 + \sigma_2) (\mathbf{r} \times \mathbf{p}) \frac{f(2\kappa r)}{r} \right\} \end{aligned} \quad (30)$$

where

$$\mathbf{p} = \frac{1}{2} (\mathbf{p}_2 - \mathbf{p}_1)$$

and

$$f(z) = C^2 \left[\left(\frac{280}{z^5} + \frac{28}{z^3} \right) K_1(z) + \frac{140}{z^4} K_0(z) \right].$$

We get thus an ^1S potential, whose largest term is

$$\sim -\frac{\mu}{M} \cdot \frac{1120}{z^6} K_1(z) \approx -\frac{112}{z^6} K_1(z).$$

This term should be compared with the largest term of the velocity independent potential $\sim -\frac{28}{z^4} K_1(z)$.

In this case, however, the terms of the first order in the velocity may be eliminated by substituting $V_{2'2'}$ with the rather similar potential $V_{2''2''}$.

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